

Applying Fractal Concept to Watershed Hydrology

S. T. Su, PhD, PE, MBA

Chief, Water Resources Section, San Francisco District

U.S. Army Corps of Engineers

333 Market Street, San Francisco, CA 94105-2197

Tel: 415 977-8854

Fax: 415 977-8687

E-mail: Shih.T.Su@spd02.usace.army.mil

Abstract

In implementing U.S. Army Corps of Engineers' Environmental Operating Principles, it becomes evident that a better, broader and insightful understanding of watershed hydrology, geomorphology, river basin patterns, coastal and stream behavior is needed for carrying out successful environmental restoration projects. One of the new concepts – fractal approach, has emerged as a promising technique to fulfill that need.

For Corps' Hydrology and Hydraulics communities, the application of fractal concept to watershed hydrology has not been attempted (at least not to the author's knowledge). Many universities have already included fractal concept and application in their hydrology and environmental curriculum. With the aid of geographic information system and advanced computer software, fractals are no longer just a geometric novelty. Fractals are now being employed as a useful tool for watershed/river basin investigations, geomorphologic studies, landscaping design and engineering applications.

This paper introduces the concept, the need and the application of the fractal approach for Corps' watershed hydrology and environmental restoration projects. It is recommended that a task force or a committee be established by the Corps to look into the fractal applications.

What is the Fractal Approach?

The concept of fractals was introduced by Mandelbrot (Mandelbrot 1967, 1982) and characterizes those objects in which properly scaled portions are identical to the original object. According to Mandelbrot, who invented the word: "I coined fractal from the Latin adjective fractus. The corresponding Latin verb frangere means, "to break" to create irregular fragments. It is therefore sensible that, in addition to "fragmented" (as in fraction or refraction), fractus should also mean "irregular," both meanings being preserved in fragment."

The fractal concept enables one to look at natural forms in the context of their functions. The fractal structure of watersheds, river basins and coastlines is a result of the interactions between water, land and sediment transport. A river network may be represented as a fractal, with a main stem that separates into smaller tributaries, which in turn separate into forks. A coastline may be represented as a fractal, with each bay or headland having its own smaller bays and headlands. It is shown that the infinite variety of watershed and river basin shapes responds to some unifying principles whose signature is found across highly different conditions (Horton 1945). The key characteristics of fractals are self-similarity and invariance of scale.

The Need and Value for Applying the Fractal Approach

A watershed is a very complex system. In the past few decades, major progress has been made toward a better understanding of how watersheds and river basins behave in terms of the rainfall-runoff transformation. The conversion of excess rainfall into surface runoff is traditionally analyzed by means of simulation models. These models assume that excess rainfall and geographical conditions over a whole watershed is uniform and that the conversion process is described by the familiar convolution integral and an instantaneous unit hydrograph for a linear system. However, in reality, the excess rainfall over a watershed is non-uniform and geographical conditions may create erratic near-field effects. As a result, major errors are introduced in the rainfall-runoff simulation models. A general conclusion that can be reached is that the watershed system is too complex for simulation models and a different approach should be explored.

The fractal concept provides a new approach and a rational basis for the arrangement of stream networks of watersheds and river basins. The infinite variety of patterns in natural watersheds and river basins suggests the existence of a basic unifying evolutionary dynamic that is responsible for pattern formation and similarity (Strahler 1957). Observations by many geomorphologists, hydrologists and hydraulic engineers confirmed their findings that the watershed and river basin landscape is a result of the interactions of geometry and functions under a minimum energy expenditure driving mechanism.

Why is the Fractal Approach Important for Watershed Hydrology and Environmental Restoration?

A good understanding of the behavior of watersheds and river basins is the prerequisite for successfully implementing the U.S. Army Corps of Engineers' Environmental Operating Principles. This is especially true for achieving the watershed sustainability.

The interplay between ecology, physics, and geometry is at the frontier of current studies of watersheds and river basins. The fractal approach considers river basins and drainage networks in light of their scaling and multiscaling properties and the dynamics responsible for their development.

The hydrology of watersheds and river basins, and prediction of their growth, demands adequate understanding of a range of temporal and spatial scales. At the core of the environmental restoration efforts is the search for the hidden order of these temporal and spatial variabilities in watersheds and river basins, despite variations in size, climate, and geology. The search concentrates on the dynamic origins of underlying mechanisms and the crucial role of self-organization.

A watershed or a river basin is a self-organizing system. The space and time distributions of water, sediment, chemicals, and biota guide the evolutions of land structure, drainage networks, habitats, and life itself. Watersheds or river basins modulate local processes and thereby alter weather, geochemical fluxes, habitats and biodiversity. They also impact global climate by the way they partition land-atmosphere-ocean fluxes and recharge to and discharge from underground aquifers. The water and energy fluxes through ocean, atmosphere and land at multiple scales in space and time interact through strongly nonlinear interactions and connections. These interactions and connections must be defined and quantified to simulate planetary environmental dynamics.

As a result of applying the fractal concept, new insights and relationships were found in both the planar and the altitude dimensions of watershed behavior. The pattern of individual river and tributary as well as the branching characteristics of the watershed system are now seen through a fractal lens. Even the response function of the watershed to a rainfall input reveals new and important underlying factors.

Application

The fractal approach is built upon the commonality of stream branching orders in response to natural phenomena. Therefore, it is applicable and highly relevant to watershed hydrology and successful environmental restoration endeavors.

Watershed Hydrology – The geomorphologic theory of the unit hydrograph (Rodriguez-Iturbe 1979) is a classical example of the application of the fractal approach to watershed hydrology. The theory interprets the runoff hydrograph as a travel time distribution to the river basin outlet, embedding the dynamics of the processes of rainfall-runoff transformation, and most importantly, geomorphology of the particular watershed. The characteristics of a watershed or river basin, as expressed as linear or other geometric

measures, can be incorporated into the formation of a representative synthetic unit hydrograph. This resultant unit hydrograph would then serve as a “computer chip” or “DNA” for a wide range of hydrologic applications.

An approach to apply the fractal concept to estimate hydrologic response is by matching suitable self-similar networks to a specific watershed, and modeling the runoff with a width-function based geomorphologic instantaneous unit hydrograph. In order to work out the identification between a specific watershed or river basin and self-similar networks that are generated by an interior generator cooperating with an exterior generator, a generalized width function is derived (Sagar 1998). Subsequently, cumulative width functions on the basis of the derived function, as well as the informational entropies are used as criteria to decide the best patterns of the two cooperating generators for the specific watershed. The next step is to calculate the runoff of this watershed or river basin as an outcome of the estimation. A power law relationship has been attempted with some success.

Coastal Studies – The coastlines are analogous to the contour lines on a topographic map of a watershed or a river basin. Because of this similitude, it is reasonable to expect that contour lines are self-similar objects that can be characterized in terms of their fractal dimensions. In a close and micro scale, one can see that the coastlines are made up of small peninsulas and bays that at the beach scale are indistinguishable from the peninsulas and gulfs at the continental scale when amplified isotropically.

To apply the fractal concept to coastal projects, one needs first to characterize the geomorphological structure of the coastal area and then relate this characterization to those hydrodynamic variables that are important for the engineering and management aspects. For example, for the San Francisco Bay, the characterization of the dendritic structure of the tidal network of the bay needs to relate to the geometry and roughness of its coastal line, and to the rivers flowing into it. (Personal Communications with Prof. Rodriguez-Iturbe 2003).

Conclusions and Recommendations

The application of the fractal concept brings a new and different perspective into the understanding and analysis of watershed hydrology. This non-traditional approach can lead to more insightful and improved design of environmental restoration projects.

Given the importance of acquiring adequate understanding of watershed hydrology for the Corps’ civil works mission, greater attention is needed to investigate the application of the fractal approach. It is recommended that a task force or a committee be established to undertake this effort.

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